Compressive-Domain Interference Cancellation

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MOTIVATION AND OVERVIEW
Compressive Sensing Architecture

\[ \mathbf{x} \xrightarrow{N} \text{measure} \quad \xrightarrow{M} \text{transmit/store} \]

\[ \text{receive} \xrightarrow{M} \text{reconstruct} \xrightarrow{N} \hat{\mathbf{x}} \]

one row of \( \Phi \)
Measurement System

\[ M \times 1 \]

measurements

\[ \Phi \]

\[ N \times 1 \]
sparse signal

\[ K \]
nonzero entries
Restricted Isometry Property

\[ M \times 1 \]
\[ \text{measurements} \]
\[ y = \Phi x \]
\[ N \times 1 \]
\[ \text{sparse signal} \]
\[ M \times N \]
\[ K \]
\[ \text{nonzero entries} \]

\[ K < M \ll N \]

\[ \Phi \] has RIP of order \( 2K \) with constant \( \epsilon \)

If there exists \( \epsilon \) s.t. for all \( 2K \)-sparse \( x \):

\[ (1 - \epsilon)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \epsilon)\|x\|_2^2 \]
Potential CS application

- Target Detection
- Target Tracking
- Signal Identification
- Signal Recovery

CS-based receiver
Target Detection

- Hypothesis Testing Problem
- Computationally simple
- Compressive domain solution ("smashed" filter)
Recover-then-filter Approach

CS Recovery (Greedy)

Interference Rejection

Target Detection

Optimization Problem (computationally expensive)

High sampling rate

Hypothesis testing problem (computationally cheap)
Compressive-domain Filtering

Desire: Maintain Signal Geometry (RIP)
COMPRESSIVE-DOMAIN FILTERING
CS Signal Acquisition

\[ M \times 1 \]

measurements

\[ y \]

\[ \Phi \]

\[ x \]

\[ N \times 1 \]
sparse signal

\[ K \times N \]
nonzero entries

\[ M < K \ll N \]

Interference

Approach: Projection—interference in the nullspace
Case I: Known Interference Support

Interference Support: $J$
Interference Coefficients: $K_J$
Signal Coefficients: $K_S$
RIP order of $\Phi$: $2K_S + K_J$

\[ P = I - \Phi_J (\Phi_J^\dagger) \]

Projection to Range($\Phi_J$)
Projection Filter, Range($\Phi_J$) in Null($P$)
RIP Conservation

If $\Phi$ has RIP of order $2K_S+K_J$ with constant $\epsilon$, i.e., exists $\epsilon$ s.t. for all $(2K_S+K_J)$-sparse $x$:

$$(1 - \epsilon)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \epsilon)\|x\|_2^2$$

then $P\Phi$ has RIP of order $2K_S$ and for all $2K_S$-sparse $\tilde{x}$:

$$\left(\frac{1-2\epsilon}{1-\epsilon}\right)\|\tilde{x}\|_2^2 \leq \|P\Phi_J c \tilde{x}\|_2^2 \leq (1 + \epsilon)\|\tilde{x}\|_2^2.$$
CS Signal Acquisition

\[ y = \Phi x \]

- \( M \times 1 \) measurements
- \( N \times 1 \) sparse signal
- \( M \times N \) nonzero entries
- \( K \) nonzero entries

Signal of Interest

\[ K < M \ll N \]
Case II: Known Signal Support

Known Signal Support: \( J \)
Interference Coefficients: \( K_J \)
Signal Coefficients: \( K_S \)
RIP order of \( \Phi \): \( K_S + 2K_J \)

\[
P = \Phi_J (\Phi_J^\dagger)
\]

Projection to Range(\( \Phi_J \))
(Rejecting everything else)
Interference Leakage Guarantee

If $\Phi$ has RIP of order $2K_S+K_J$ with constant $\epsilon$, i.e., exists $\epsilon$ s.t. for all $(2K_S+K_J)$-sparse $x$:

$$(1 - \epsilon)\|x\|^2 \leq \|\Phi x\|^2 \leq (1 + \epsilon)\|x\|^2$$

Then the interference is attenuated at least by:

$$\|P\Phi_{J^c} \tilde{x}\|^2 \leq \frac{\epsilon^2(1 + \epsilon)}{(1 - \epsilon)^2}\|\tilde{x}\|^2$$
Remarks on $\Phi_J(\Phi_J^\dagger)$

- $\Phi_J(\Phi_J^\dagger)$ not the only possible choice
  - Any projection to $\text{Range}(\Phi_J)$ works
  - Convenient if access to columns of $\Phi_J$ not explicit
  - Efficient implementation

- $P$ is rank/dimension reducing by $K_J$ or $M-K_J$
  - $\text{Rank}(P\Phi)=M-K_J$ or $K_J$
  - Subsequent computation more efficient
EXPERIMENTS
Oracle Reconstruction

- Oracle aware of interference and signal support
- Reconstruction using the pseudoinverse on that support
- Rejection of the interference coefficients post-reconstruction
- Reconstruction error is the $\ell_2$ error on the signal of interest
Recover-then-filter Reconstruction

- Reconstruction (CoSaMP) not aware of signal vs. interference
- Reconstruction recovers both signal and interference
- Rejection of the interference coefficients post-reconstruction
- Reconstruction error is the $\ell_2$ error on the signal of interest
Interference-Aware Reconstruction

- Modified reconstruction (CoSaMP) aware of interference support
- Reconstruction recovers both signal and interference
- Rejection of the interference coefficients post-reconstruction
- Reconstruction error is the $\ell_2$ error on the signal of interest
Filter-then-recover Reconstruction

- CS-domain filtering to reject interference
- Reconstruction (CoSaMP) only recovers the signal
- No need to reject coefficients post-reconstruction
- Reconstruction error is the $\ell_2$ error on the signal of interest
Results—Error Performance

Recovered SNR (dB)

Oracle
Cancel–then–recover
Modified recovery
Recover–then–cancel

$K_I/K_S$
Results—Computation

Recovery Time (s) vs. $K_I/K_S$

- **Cancel-then-recover** (black dashed line with stars)
- **Modified recovery** (red dashed line with circles)
- **Recover-then-cancel** (blue solid line with plus signs)
Concluding Remarks

- Reconstruction is usually **not required** in applications.
- Steps towards **compressive-domain signal processing**.
- Compressive-domain processing can be **more efficient**.
- Filtering is an **essential** signal processing operation.
- Preliminary results. Much more on the way.
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