Compressive-Domain Interference Cancellation

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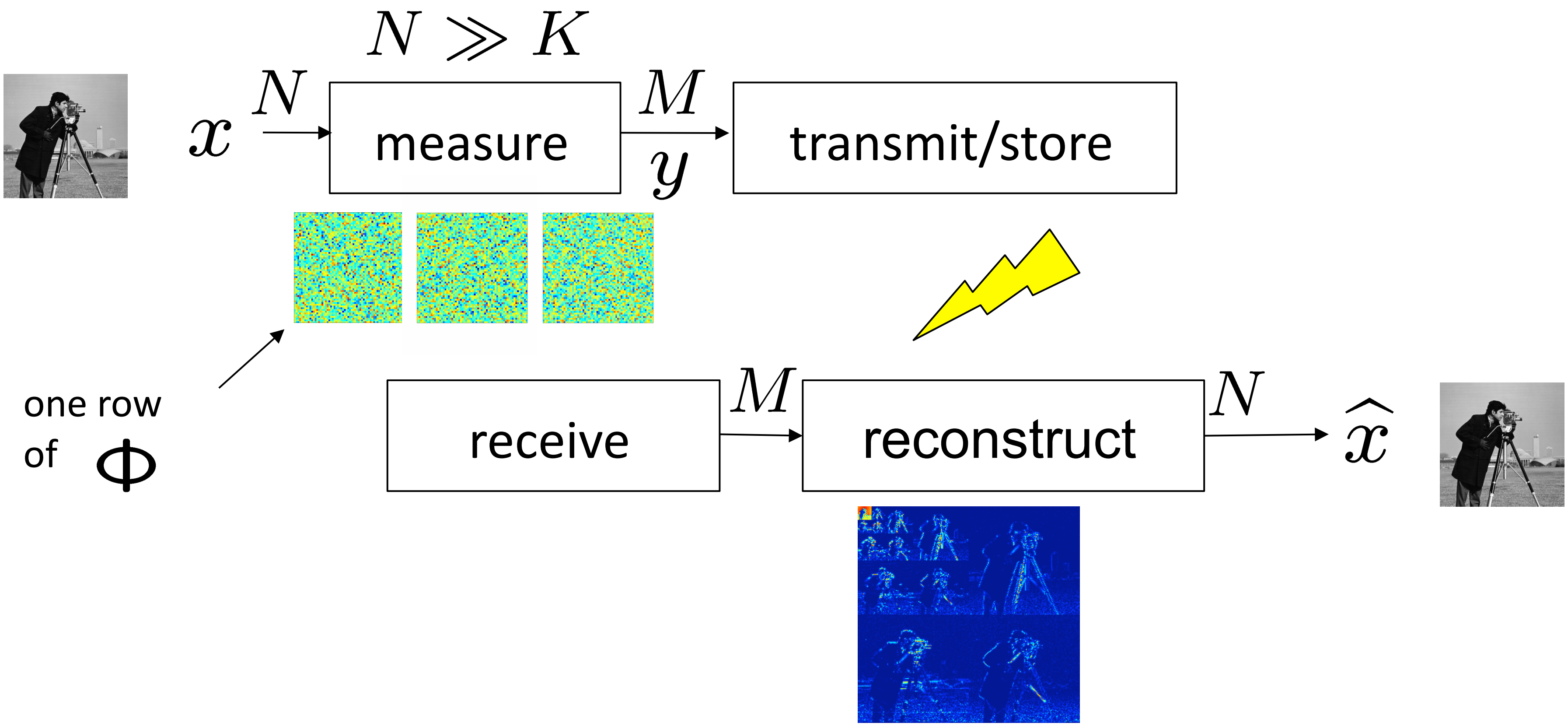
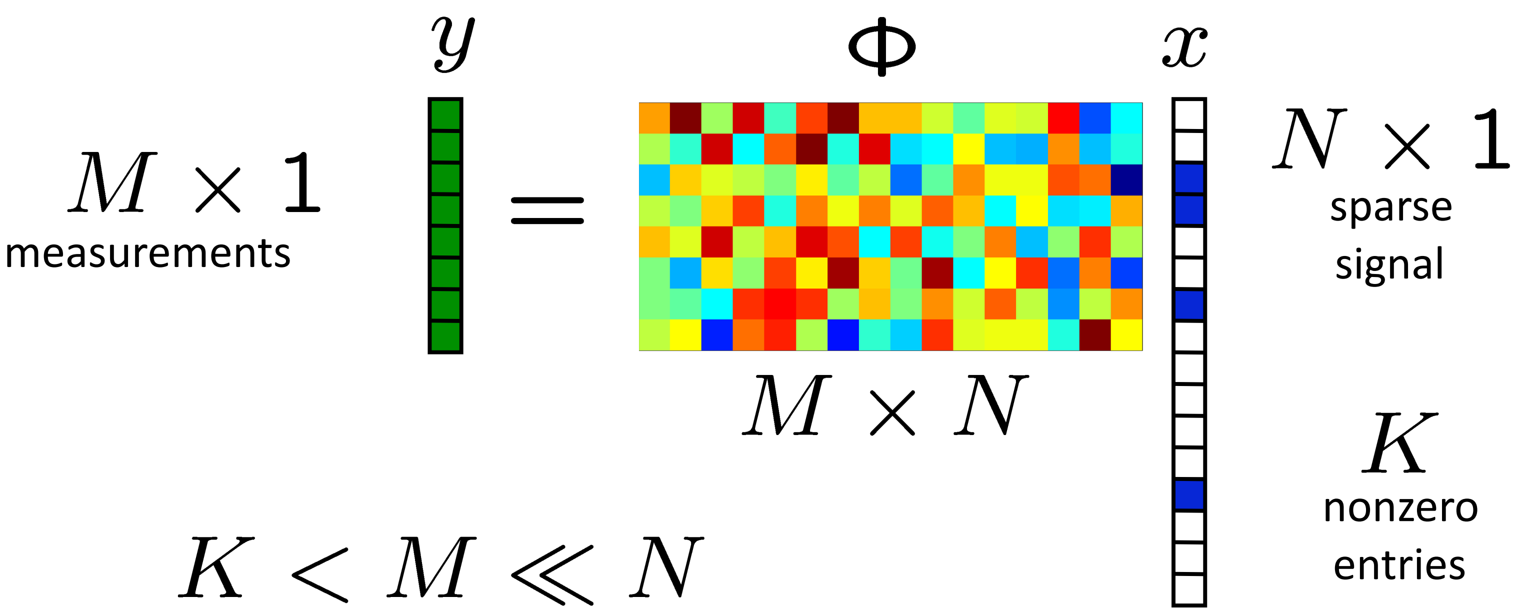
MOTIVATION AND OVERVIEW

Compressive Sensing Architecture

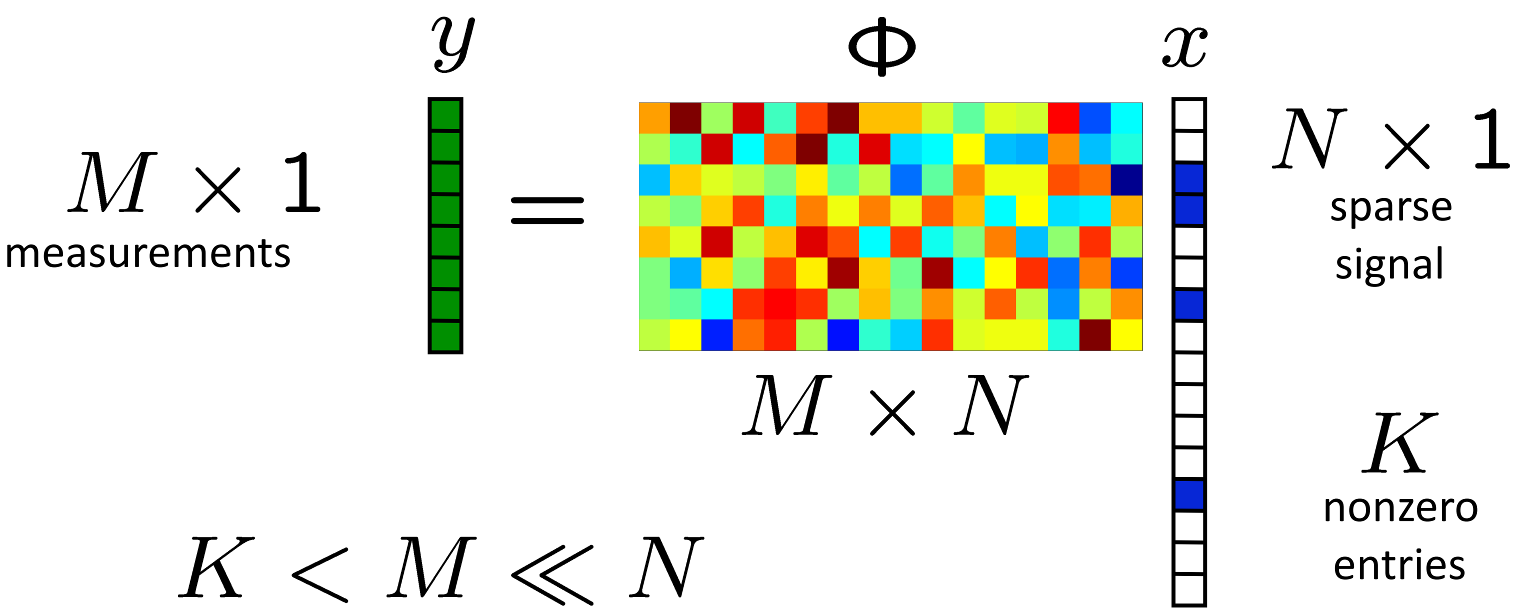


Measurement System



$$K < M \ll N$$

Restricted Isometry Property



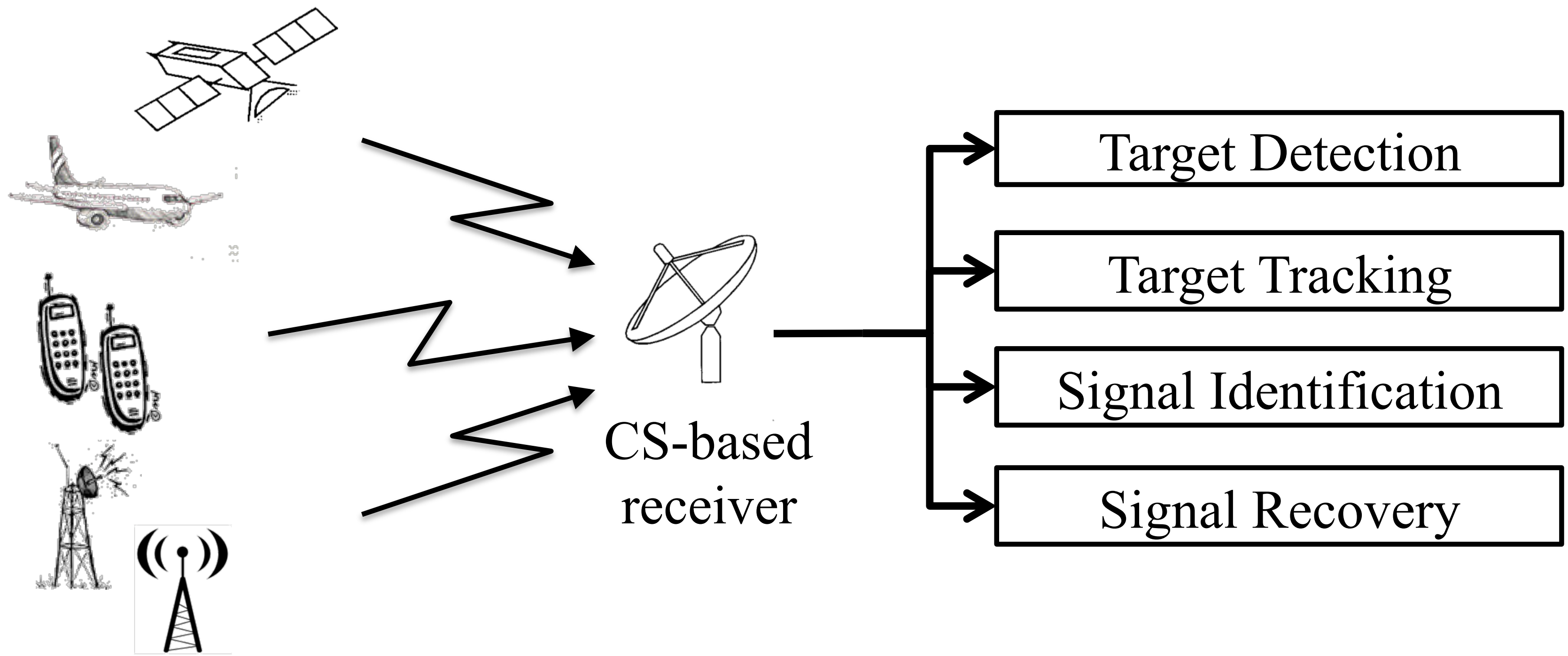
$$K < M \ll N$$

Φ has RIP of order $2K$ with constant ϵ

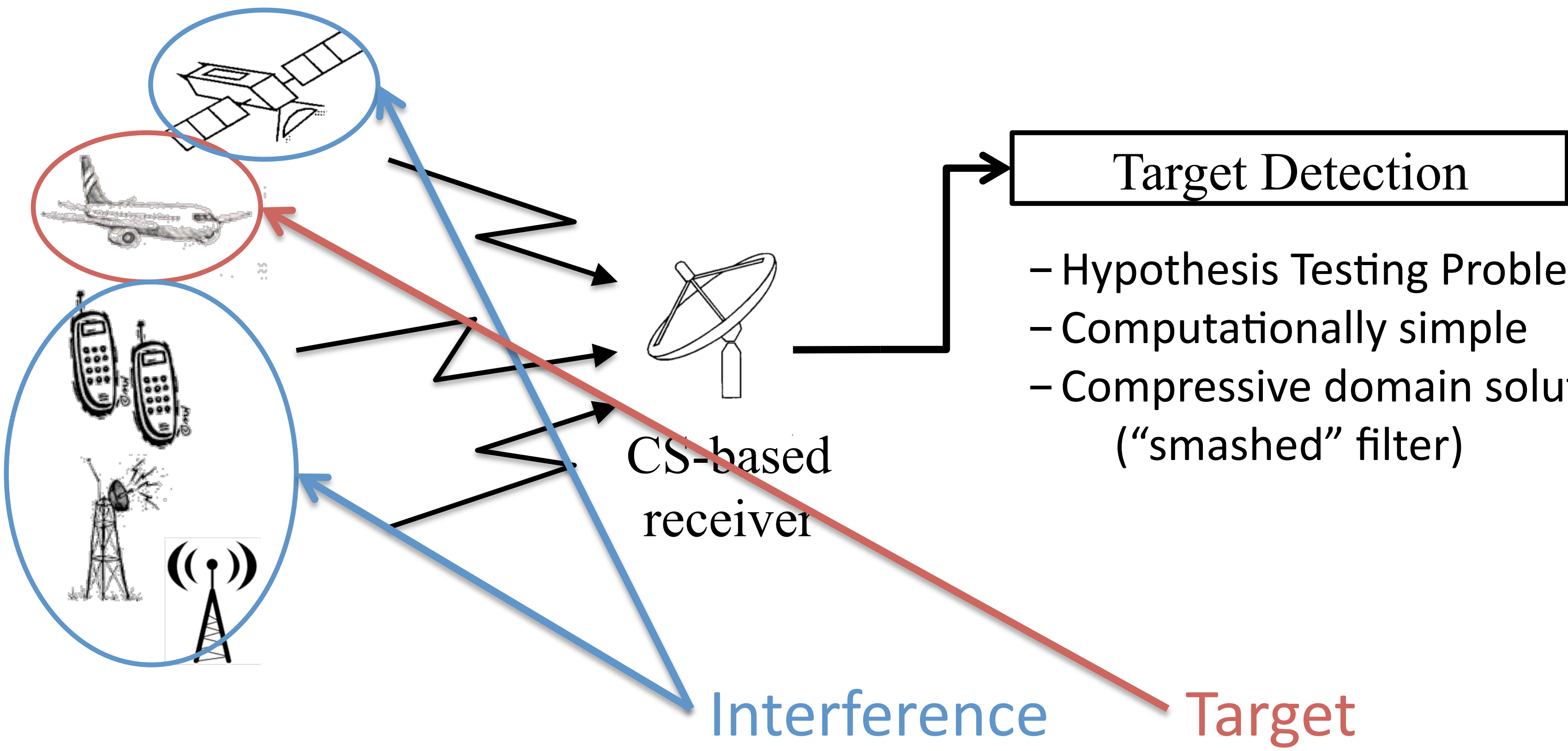
If there exists ϵ s.t. for all 2K-sparse x:

$$(1-\epsilon)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1+\epsilon)\|x\|_2^2$$

Potential CS application

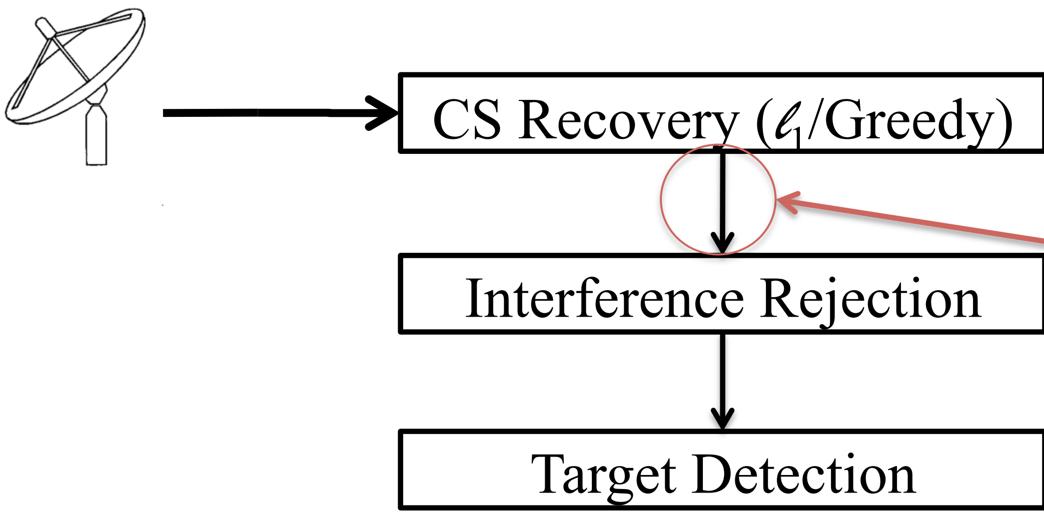


Target Detection



- Hypothesis Testing Problem
- Computationally simple
- Compressive domain solution ("smashed" filter)

Recover-then-filter Approach

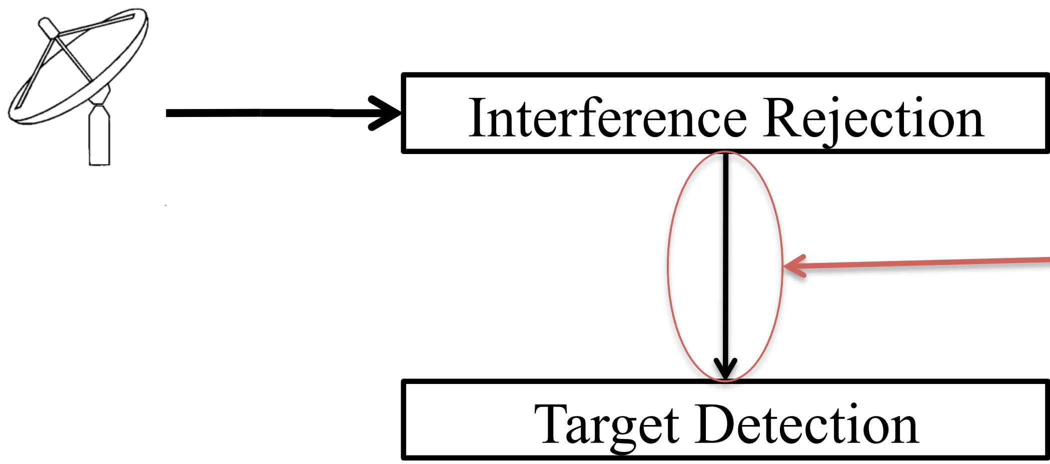


Optimization Problem
(computationally expensive)

High sampling rate

Hypothesis testing problem
(computationally cheap)

Compressive-domain Filtering



Compressive Domain Filter

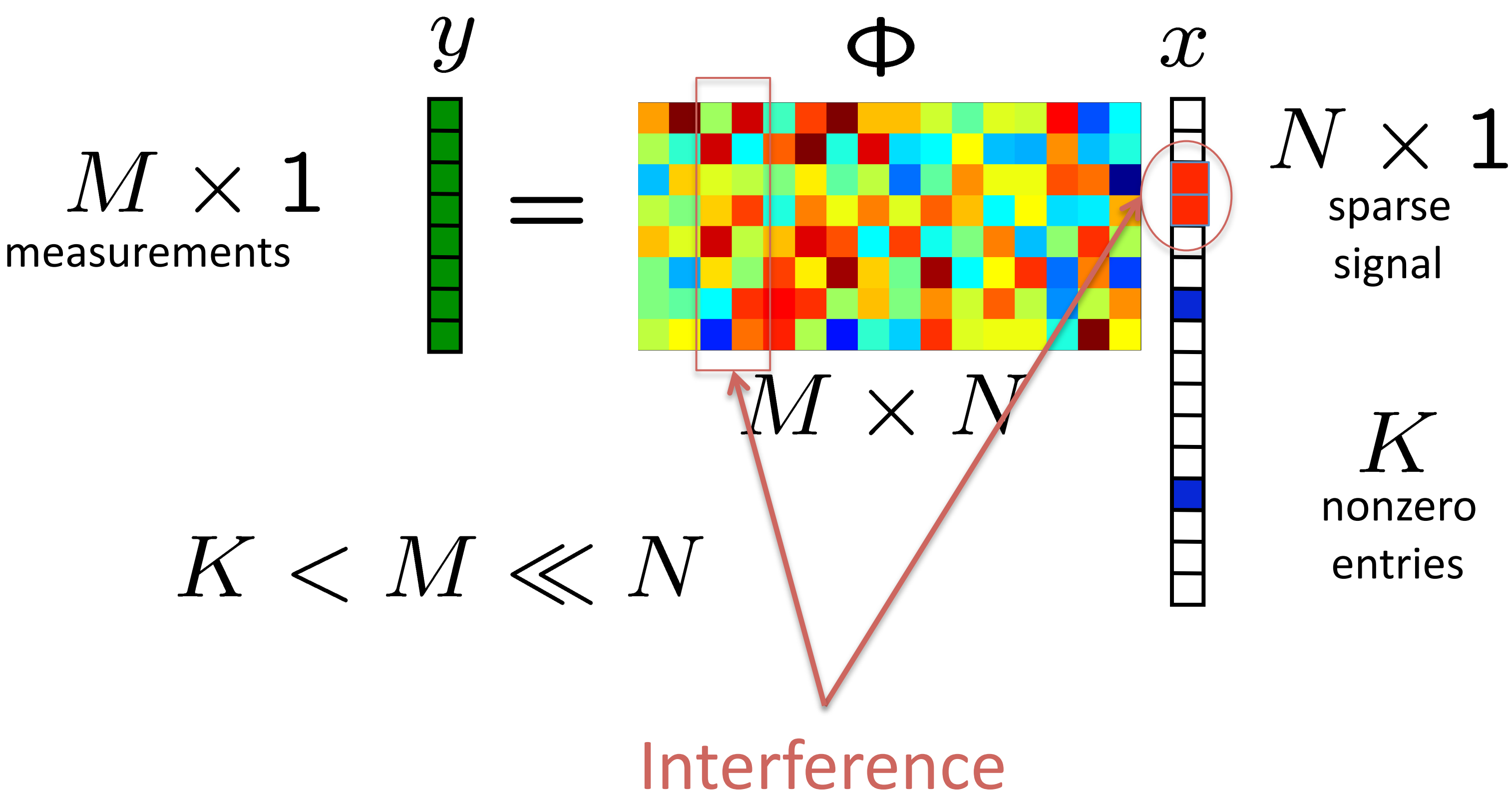
Low sampling rate

Hypothesis testing
(Compressive domain)

Desire: Maintain Signal Geometry (RIP)

COMPRESSIVE-DOMAIN FILTERING

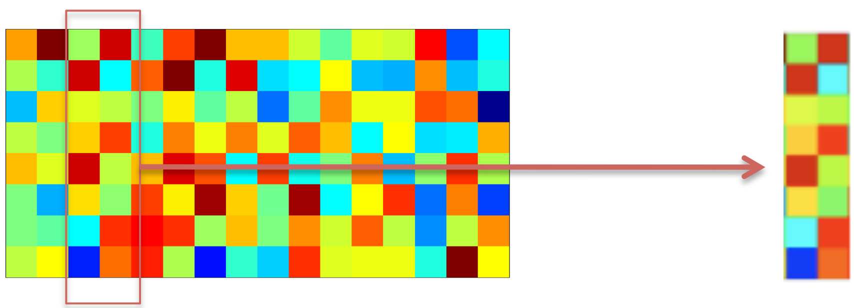
CS Signal Acquisition



Approach: Projection—interference in the nullspace

Case I: Known Interference Support

Φ Φ_J



$$\underbrace{P = I - \underbrace{\Phi_J(\Phi_J^{\dagger})}_{\text{Projection to Range}(\Phi_J)}}_{\text{Projection Filter, Range}(\Phi_J)\text{ in Null}(P)}$$

Interference Support: J
Interference Coefficients: K_J
Signal Coefficients: K_S
RIP order of Φ: $2K_S+K_J$

RIP Conservation

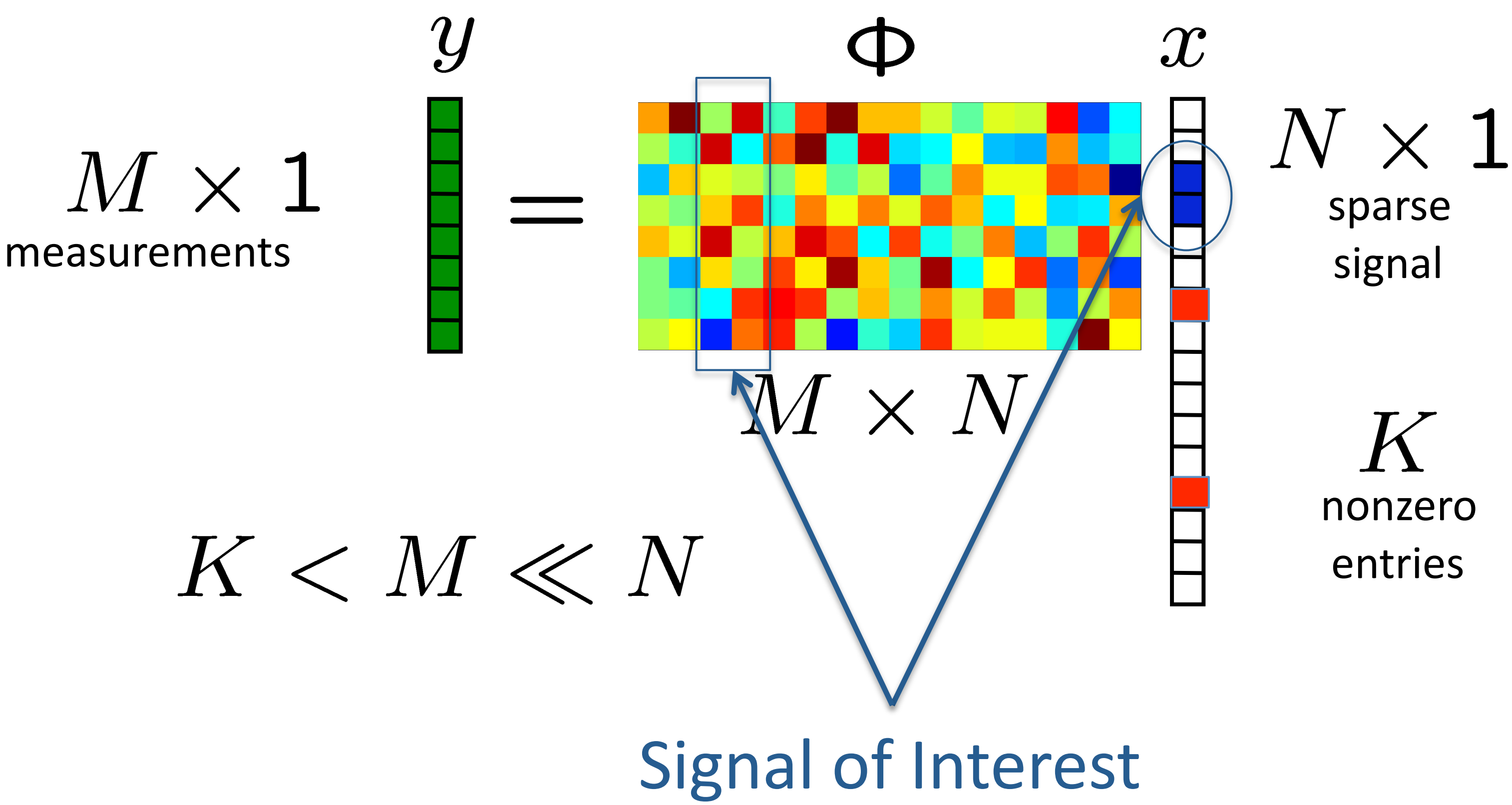
If Φ has RIP of order $2K_S$+K_J with constant ϵ,
i.e., exists ϵ s.t. for all ($2K_S$+K_J)-sparse x:

$$(1-\epsilon)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1+\epsilon)\|x\|_2^2$$

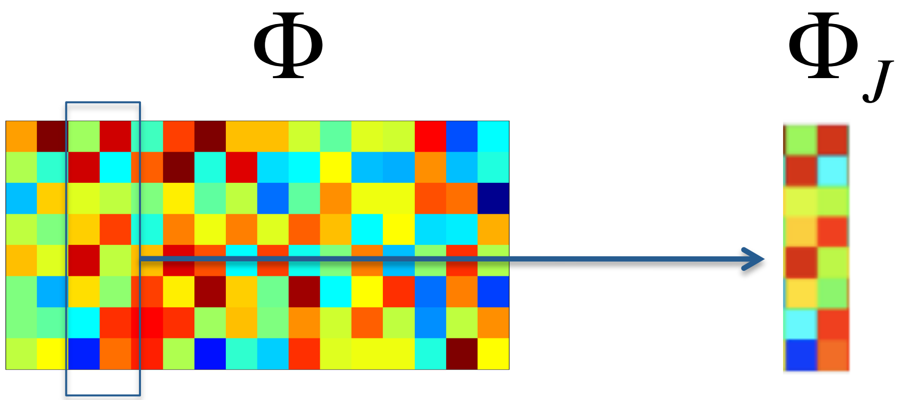
then $P\Phi$ has RIP of order $2K_{S,}$ and for all $2K_S$-sparse $\widetilde{x}$:

$$\left(\frac{1-2\epsilon}{1-\epsilon}\right)\|\widetilde{x}\|_2^2 \leq \|P\Phi_{J^c}\widetilde{x}\|_2^2 \leq (1+\epsilon)\|\widetilde{x}\|_2^2$$

CS Signal Acquisition



Case II: Known Signal Support



$$P = \underbrace{\Phi_J(\Phi_J^{\dagger})}_{\substack{\text{Projection to} \\ \text{Range}(\Phi_J) \\ \text{(Rejecting everything else)}}}$$

Known Signal Support: J
Interference Coefficients: K_J
Signal Coefficients: K_S
RIP order of Φ: $K_S + 2K_J$

Interference Leakage Guarantee

If Φ has RIP of order $2K_S$+K_J with constant ϵ,
i.e., exists ϵ s.t. for all ($2K_S$+K_J)-sparse x:

$$(1-\epsilon)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1+\epsilon)\|x\|_2^2$$

Then the interference is attenuated at least by:

$$\|P\Phi_{J^c}\widetilde{x}\|_2^2 \leq \frac{\epsilon^2(1+\epsilon)}{(1-\epsilon)^2}\|\widetilde{x}\|_2^2$$

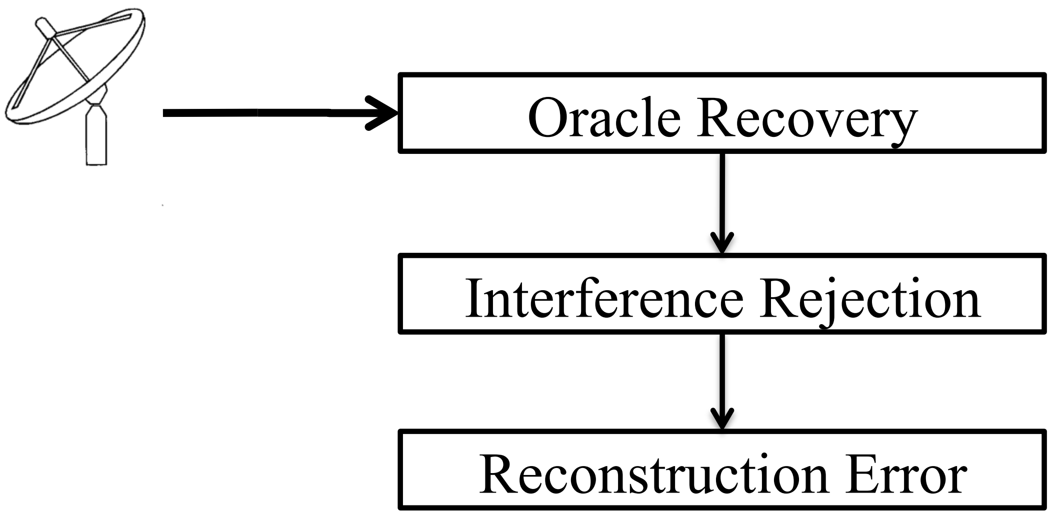
Remarks on $\Phi_J(\Phi_J^\dagger)$

- $\Phi_J(\Phi_J^\dagger)$ not the only possible choice
  - Any projection to Range(Φ_J) works
  - Convenient if access to columns of Φ_J not explicit
  - Efficient implementation

- P is rank/dimesion reducing by K_J or M-K_J
  - Rank($P\Phi$)=M-K_J or K_J
  - Subsequent computation more efficient

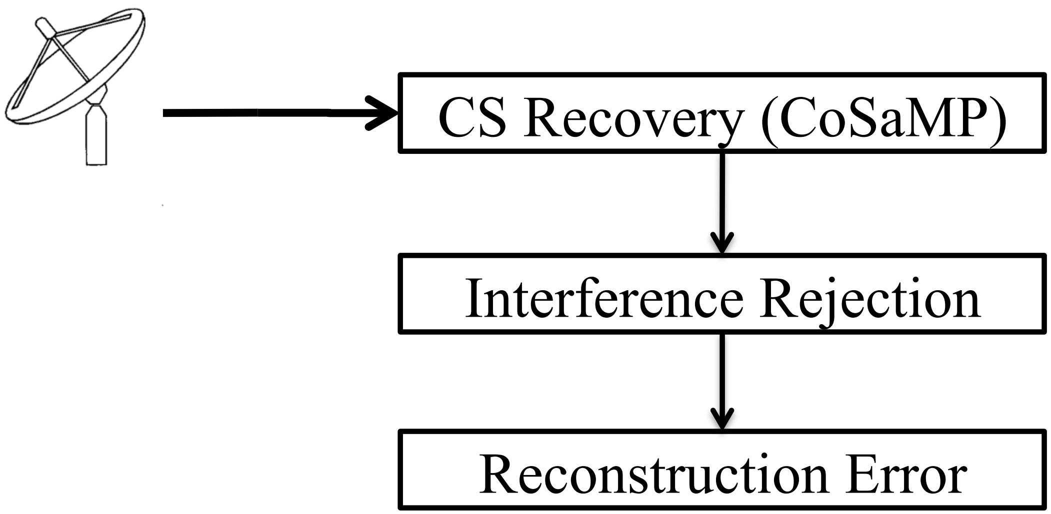
EXPERIMENTS

Oracle Reconstruction



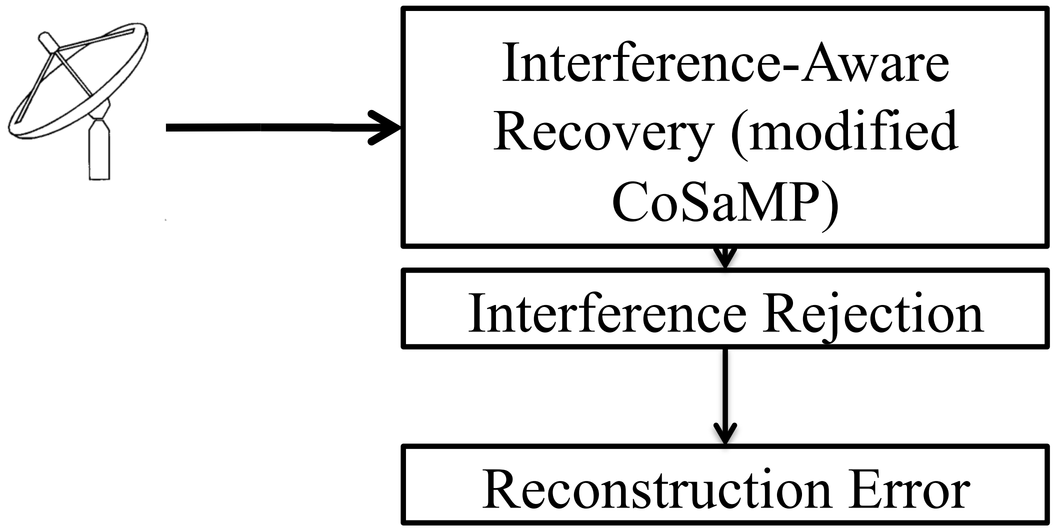
- Oracle aware of interference and signal support
- Reconstruction using the pseudoinverse on that support
- Rejection of the interference coefficients post-reconstruction
- Reconstruction error is the ℓ_2 error on the signal of interest

Recover-then-filter Reconstruction



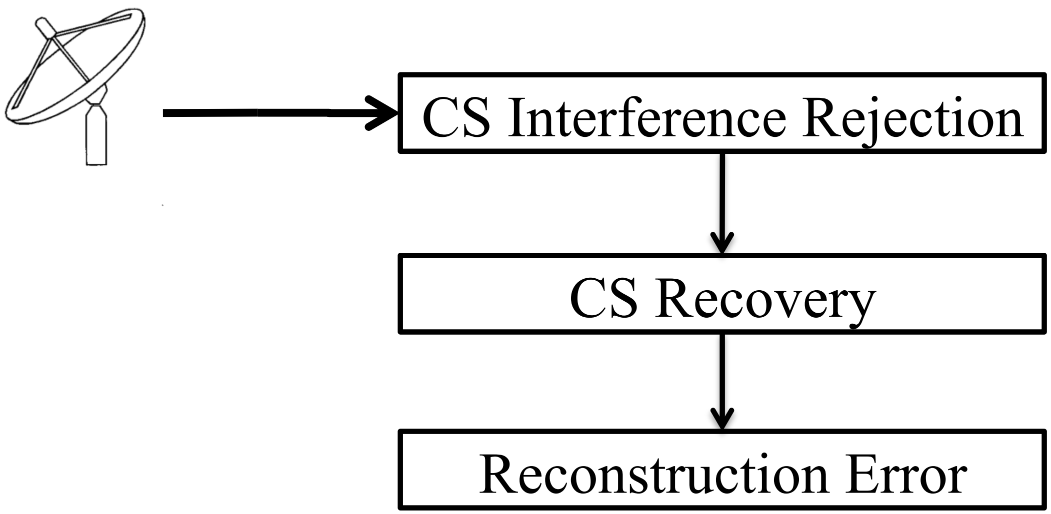
- Reconstruction (CoSaMP) not aware of signal vs. interference
- Reconstruction recovers both signal and interference
- Rejection of the interference coefficients post-reconstruction
- Reconstruction error is the ℓ_2 error on the signal of interest

Interference-Aware Reconstruction



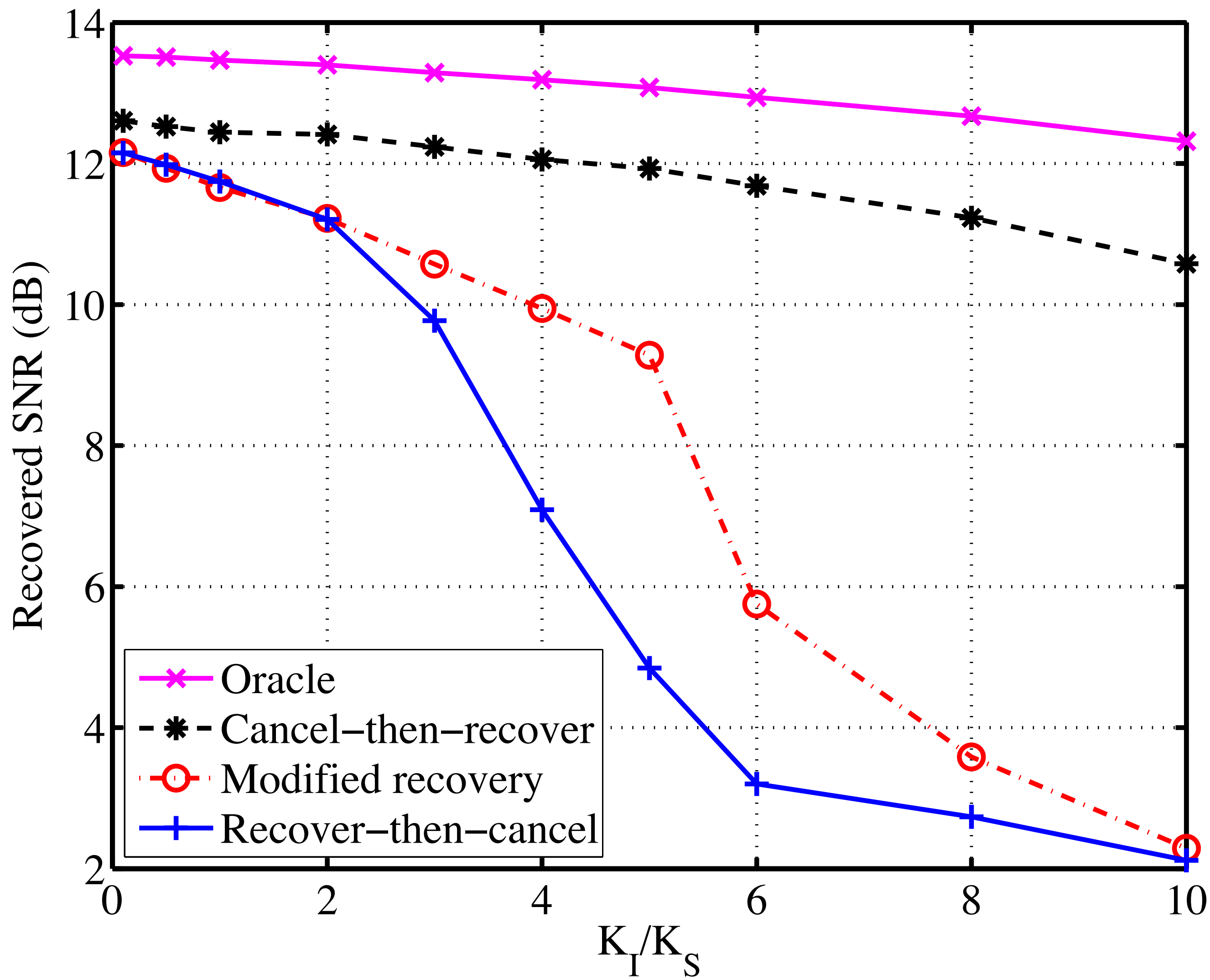
- Modified reconstruction (CoSaMP) aware of interference support
- Reconstruction recovers both signal and interference
- Rejection of the interference coefficients post-reconstruction
- Reconstruction error is the ℓ_2 error on the signal of interest

Filter-then-recover Reconstruction

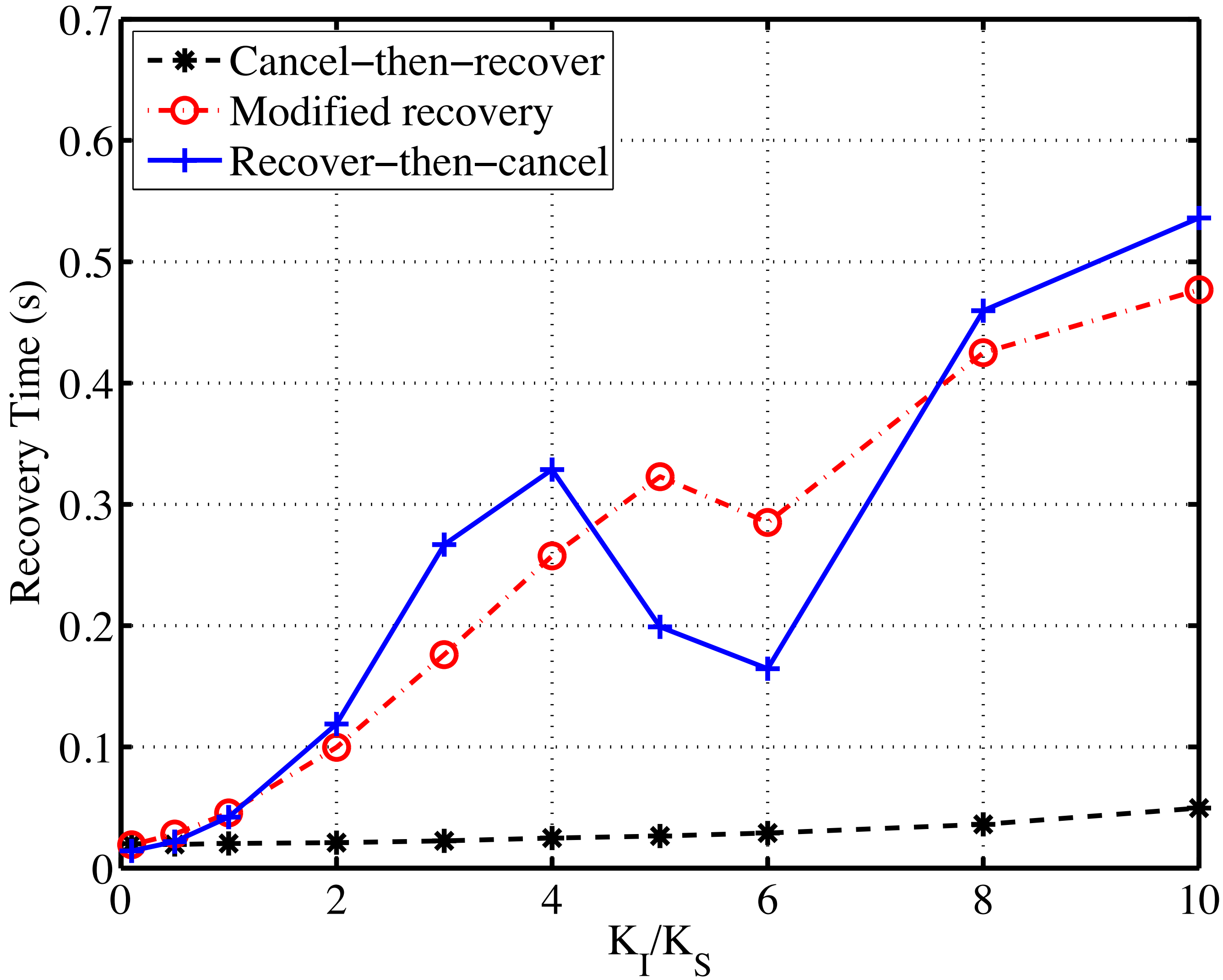


- CS-domain filtering to reject interference
- Reconstruction (CoSaMP) only recovers the signal
- No need to reject coefficients post-reconstruction
- Reconstruction error is the ℓ_2 error on the signal of interest

Results—Error Performance



Results—Computation



Concluding Remarks

- Reconstruction is usually not required in applications.
- Steps towards compressive-domain signal processing.
- Compressive-domain processing can be more efficient.
- Filtering is an essential signal processing operation.
- Preliminary results. Much more on the way
- Questions: petrosb@merl.com