HIGH RESOLUTION SAR IMAGING USING RANDOM PULSE TIMING

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ABSTRACT

Synthetic Aperture Radar (SAR) is a fundamental technology with significant impact in remote sensing applications. SAR relies on the motion of the radar platform to synthesize a large aperture, and achieve high resolution imaging of a large area. However, current strip-map SAR designs, relying on uniform pulsing, suffer from a fundamental trade-off between the azimuth resolution and the range coverage length. In this paper we overcome this trade-off using a randomized pulsing scheme combined with non-linear compressive sensing (CS) reconstruction. Our experimental results demonstrate significant improvement in the azimuth resolution using the proposed approach, without compromise on the range length of the imaged area.

***Index Terms*—** Synthetic Aperture Radar(SAR), high-resolution imaging, randomized pulse timing, Compressive Sensing(CS)

1. INTRODUCTION

Synthetic Aperture Radar (SAR) is a high resolution radar imaging technology with significant impact in remote sensing. SAR exploits the motion of the radar platform (such as a vehicle, a satellite or a plane) to synthesize a large virtual aperture that can image large areas with high resolution. Classical SAR transmits pulse signals (chirps) at a uniform rate. The corresponding echoes, reflected from the region of interest, are processed to reconstruct a two-dimensional image. The image resolution on the axis perpendicular to the motion of the platform (*range*) is determined by the bandwidth of the transmitted pulse, while the resolution along the axis of motion (*azimuth*) depends on the synthetic aperture size due to the Doppler effect [1].

However, classical strip-map SAR exhibits a fundamental trade-off between the azimuth resolution and the range coverage length, stemming from the need to separate the pulse transmission from the echo reception. Most SAR systems use the same antenna for the transmission of the pulse and the reception of the echo. While a pulse is transmitted the radar cannot receive the reflection of another pulse from the ground. Thus, the time interval between two transmitted pulses should be large enough or the pulse repetition frequency (PRF) be low enough such that the whole reflection from ground due to one pulse can be acquired. Otherwise the transmitted pulses will interfere with the reception of the received echoes and lose range coverage because of missing data. On the other hand, the Doppler bandwidth requires a minimum PRF to avoid ambiguity according to the Nyquist sampling theorem. If the PRF is too low, it will cause azimuth ambiguities because of aliasing. Thus, to increase the SAR azimuth resolution it is necessary to increase the acquisition PRF.

In recent years, the development of compressive sensing (CS) [2] has had great impact in sensing applications, including radar imaging [3–7]. CS fundamentally revisits signal acquisition and allows robust reconstruction of signals using a significantly smaller number of measurements compared to their Nyquist rate. This sampling rate reduction is achieved by using randomized measurements, improved signal models and non-linear reconstruction algorithms. Recent progress on compressive sensing radar has been made by [4–6], but most results consider point targets or simplified experiment setups—generally not applicable in radar imaging systems. A number of challenges still exist and need to be resolved to apply CS to radar imaging, such as developing appropriate sparsity models of radar images and managing computational complexity [3, 7]. With this work we attempt a small step towards addressing those challenges.

In this paper, we propose a randomized pulsing scheme for SAR, combined with compressive sensing reconstruction, to significantly improve the trade-off between range length and azimuth resolution and overcome the fundamental limitations of classical pulsing. Our work combines two distinct modifications:

1. Randomized pulse timing with high *average* PRF to achieve high azimuth resolution and robustness to the missing data due to the pulse transmission.
2. Iterative CS reconstruction algorithms to handle missing data and overlapping echoes, and to incorporate image models.

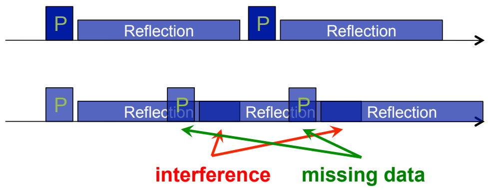
To validate our approach, we provide experimental results using simulated acquisition with both classical uniform and randomized pulsing, and demonstrate that we significantly improve the azimuth resolution. While we focus the experiments in one particular mode of SAR operation—namely, strip-map SAR—the method is fundamentally applicable to other modes, such as spotlight mode.

In the next section we provide a brief overview on SAR acquisition, with some emphasis on the pulse timing. Section 3 describes how randomized pulse timing can be used to increase SAR azimuth resolution. In Section 4 we briefly examine CS-inspired image formation algorithms to reconstruct from the acquired echoes. Section 5 provides experimental results that confirm and validate our approach. We conclude with some discussion in Section 6.

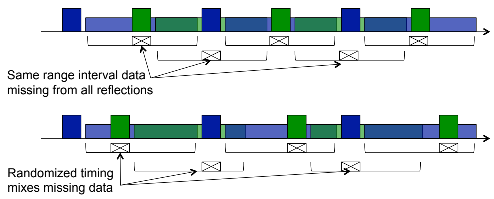
2. BACKGROUND

To image a swath of land, SAR systems transmit pulses, usually linear chirps, at a uniform rate. The pulses are reflected from the ground, and the reflections (echoes) are recorded and used to reconstruct the ground reflectivity.

Classical SAR pulse timing is depicted on the top of Fig. 1(a). The duration of each reflection depends on the length of the area of interest in the range direction. The reflection (echo) of the pulse is effectively a convolution of the transmitted pulse with the part of the ground illuminated by the pulse. The time difference for the pulse to travel to the farthest ground points compared to the nearest ones determines the length of the reflection. Illuminating larger range areas produces longer echoes. Therefore, increasing the range length



(a) Classical pulsing



(b) Incoherent pulsing and randomized timing

Fig. 1. SAR Pulse timing. (a) Classical SAR pulsing. To avoid overlaps the PRF is limited by the length of the reflection. (b) Randomized pulse timing. If the pulsing timings are not randomized, the same part of the reflection will always be missing. Randomizing the pulse timing allows us to image all parts of the region.

of the image requires decreasing the PRF. Otherwise, multiple received echoes would interfere with each other and with the transmitting pulses, causing overlapping and missing data in reflections, as shown on the bottom of Fig. 1(a). Lower PRF, in turn, decreases the Doppler bandwidth that can be sampled, and, therefore, the achievable azimuth resolution.

The SAR data acquisition process can be described overall as a linear operation:

$$\mathbf{y} = \mathbf{\Phi}(\mathbf{x}) + \mathbf{n}, \tag{1}$$

where $\mathbf{y}$ denotes the received radar echoes, $\mathbf{x}$ denotes the reflectivity of the imaged area, $\mathbf{\Phi}$ models the SAR acquisition function depending on the radar parameters, and $\mathbf{n}$ is measurement noise.

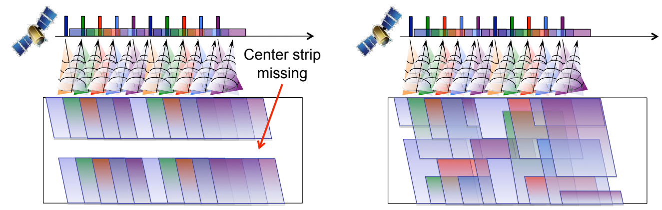
The goal of the image formation process is to determine the signal of interest $\mathbf{x}$ from the radar echoes $\mathbf{y}$ given the acquisition function $\mathbf{\Phi}$. In other words, it solves an inverse problem. If the acquisition function $\mathbf{\Phi}$ is invertible, an obvious choice is to use the inverse or the pseudoinverse of $\mathbf{\Phi}$ to determine $\mathbf{x}$:

$$\hat{\mathbf{x}} = \mathbf{\Phi}^{\dagger}(\mathbf{y}). \tag{2}$$

In practical SAR systems, $\mathbf{\Phi}$ is difficult to model accurately and the inversion can be computationally expensive. Typically, SAR image formation is achieved using one of the well-established algorithms such as the Range-Doppler Algorithm (RDA) or the Chirp Scaling Algorithm (CSA) to approximate the inversion problem [1].

3. RANDOMIZED PULSE TIMING

Our main goal in this paper is to increase the average PRF in SAR systems without compromising the acquired range length. As we discuss in Sec. 2, we need to address two main issues when increasing the PRF. The first issue is the interference between overlapping reflected echoes from two different pulses. The second issue is the data missing when a pulse is transmitted concurrently with an echo reception.



(a) Uniform pulse timing (b) Randomized pulse timing

Fig. 2. Ground coverage with high PRF. (a) With uniform pulse timing the same range length is always missing, and cannot be recovered. (b) With randomized pulse timing the missing data is spread uniformly in the range length, and the ground is well covered.

In order to avoid interference of two overlapping echoes, one option is to use incoherent[1] or orthogonal pulses, such as an upchirp followed by a downchirp. Thus, the interference is minimized and an iterative reconstruction algorithm can separate overlapping reflections effectively and efficiently using pulse compression. Figure 1(b) shows an example, in which two different pulses are denoted using different colors. Note that for more frequent pulsing, in which case three or more reflections might overlap, more than two different pulse shapes are necessary to be able to separate the reflections.

Even if the pulse reflections can be efficiently separated, the interference from the transmitted pulse causes missing data in the reflection. If the pulsing timing is uniform, the missing data are always located in the same position of the reflection, as shown on the top of Fig. 1(b). Iterative algorithms are in general able to handle missing data but even the best approach (including CS methods) will fail if the same range locations are always missing. The resulting image will have an unrecoverable region at that range interval.

To be able to reconstruct in the presence of missing data, we randomize the pulse timing, ensuring that the missing data from every reflection corresponds to a different range interval. The timing of this approach is demonstrated at the bottom of Fig. 1(b). Since the ground coverage of adjacent reflections overlap significantly in azimuth, we can make use of the information redundancy to reconstruct an image with both high azimuth resolution and large range length by suitable iterative reconstruction algorithms.

Figure 2 demonstrates how randomizing the pulse timing distributes the missing data with respect to the ground coverage in strip-map SAR. The figure demonstrates how missing data from a reflected pulse corresponds to missing range intervals on the ground. Uniform pulse timing, depicted in Fig. 2(a), results to the same ground range always being missed. On the other hand, randomized pulse timing, shown in Fig. 2(b), uniformly distributes the missing data, facilitating reconstruction.

This approach is inspired by the CS paradigm. The randomization ensures that the linear measurements are incoherent and fully capture the signal information. Thus the measurements can be inverted by the non-linear reconstruction process using the signal model or the appropriate regularization to recover the acquired signal. For example, assuming that the image to be reconstructed is

[1]The notion of coherence has different meaning in the radar and the CS community. In the radar community, the term is usually associated with the phase properties of the signals, such as the pulse, the echo, or the formed image. In CS, coherence refers to the inner product between two different signals, dictionary elements, or measurement vectors. We use the CS notion of coherence exclusively in this paper.

sparse in some basis, such as the wavelet basis, we can reconstruct the image from the random missing measurements using sparsity-enforcing regularization.

To manage the reconstruction computational complexity, we discretize the random timing for the pulse transmission. Specifically, the pulse timings are selected randomly from a dense uniform grid of possible transmission times, using a Bernoulli process with probability $\beta < 1$. If the uniform grid intervals are at frequency $\alpha f_a, \alpha > 1/\beta$, where f_a denotes the PRF of classical pulse timing, the resulting average PRF is $\alpha\beta f_a$—an $\alpha\beta$ times increase.

4. IMAGE RECONSTRUCTION

To reconstruct from the acquired pulses, we assume that we have access to the accurate model $\mathbf{\Phi}$ and we assume the noise is negligible. While we believe our model is robust to model uncertainties and high noise conditions, we defer a more careful examination of robustness to a later publication. Even under these assumptions, the reconstruction problem is not straightforward, because of the overlapping echoes and the missing data. Furthermore, we desire to enforce a signal model, such as sparsity in some basis. Thus, using iterative reconstruction algorithms is necessary to recover the ground reflectivity.

In this paper we use gradient descent followed by thresholding, the basic workhorse for a large number of such algorithms. Specifically, we formulate the problem as a minimization of a cost function which measures the data fidelity, subject to a sparsity constraint which enforces a signal model. The iterative minimization algorithm tries to determine the ground reflectivity $\widehat{\mathbf{x}}$ such that the corresponding acquired data $\mathbf{\Phi}(\widehat{\mathbf{x}})$ are close to the actual acquired data $\mathbf{y}$ and a basis transform $g(\widehat{\mathbf{x}})$ of the reflectivity is sparse. In other words, the ground reflectivity solves the following optimization:

$$\widehat{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{\Phi}(\mathbf{x}) - \mathbf{y}||_2^2 \text{ s.t. } \|g(\mathbf{x})\|_0 < N, \tag{3}$$

where N controls the sparsity of the formed image. For the remainder of this paper, we use the wavelet transform for $g(\cdot)$.

At iteration k, the iterative algorithm maintains an estimate of the ground reflectivity, $\mathbf{x}$. This is used to compute the gradient to the cost function, $\mathbf{\Phi}^H(\mathbf{\Phi}(\mathbf{x}_k) - \mathbf{y})$, where $\mathbf{\Phi}^H(\cdot)$ is the adjoint of the acquisition model $\mathbf{\Phi}(\mathbf{x})$. The algorithm updates the estimate by following the gradient, with stepsize τ, towards minimizing the cost function, and soft-thresholding the result. The procedure is iterated for K steps. The algorithm, which is very similar to the Fixed Point Continuation (FPC) [8], is summarized as follows:

1. Initialize $\mathbf{x}_0 = \mathbf{0}$,
2. FOR $k = 1 : K$
   gradient search:
   $$\tilde{\mathbf{x}}_k = \mathbf{x}_{k-1} - \tau\mathbf{\Phi}^H(\mathbf{\Phi}\mathbf{x}_{k-1} - \mathbf{y})$$
   soft-thresholding wavelet coefficients:
   $$\begin{aligned} \mathbf{c} &= \mathbf{W}(\tilde{\mathbf{x}}_k) \\ \mathbf{b} &= \mathcal{T}_N(\mathbf{c}) = \max(|\mathbf{c}| - \epsilon_N(\mathbf{c}), 0) \circ e^{-j\angle\mathbf{c}} \\ \mathbf{x}_k &= \mathbf{W}^{-1}(\mathbf{b}) \end{aligned}$$
   END
3. Output $\mathbf{x} = \mathbf{x}_K$.

In the algorithm, $\mathbf{W}(\cdot)$ and $\mathbf{W}^{-1}(\cdot)$ denote the wavelet transform, $\angle\cdot$ the element-wise phase operator, $\circ$ the element-wise multiplication, $j = \sqrt{-1}$, and $\epsilon_N(\mathbf{c})$ is the magnitude of the $(N+1)^{th}$ largest in magnitude coefficient of $\mathbf{c}$.

Efficient execution of the algorithm relies on efficient implementation of the forward and the adjoint operators, $\mathbf{\Phi}(\cdot)$ and $\mathbf{\Phi}^H(\cdot)$ respectively. In classical pulse timing, the CSA can be used to provide such an efficient implementation [9]. Similarly, the CSA can also be the basis of an efficient implementation in the case of randomized pulse timing, provided we properly account for the missing data and the overlapping echoes.

The missing data can be easily accommodated by not taking them into account when evaluating the cost function and its gradient in the algorithm. This is equivalent to subsampling the acquisition system output using a sampling operator $\mathbf{\Sigma}$, which selects only the data we have at our disposal. The resulting system is $\mathbf{\Sigma\Phi}$ and its adjoint is equal to $\mathbf{\Phi}^H\mathbf{\Sigma}^H$, both very efficient to implement.

The overlapping echoes can be accommodated using the linearity of the acquisition model and the distributivity of the adjoint operator, i.e., using $(\mathbf{\Phi}_1 + \mathbf{\Phi}_2)^H = \mathbf{\Phi}_1^H + \mathbf{\Phi}_2^H$. Thus, the forward and the adjoint can be implemented as if the echoes were separated, followed by summation of the output of the operators. The implementation of the individual operators using the CSA is very similar to [9]; more detailed description is deferred to a future publication.

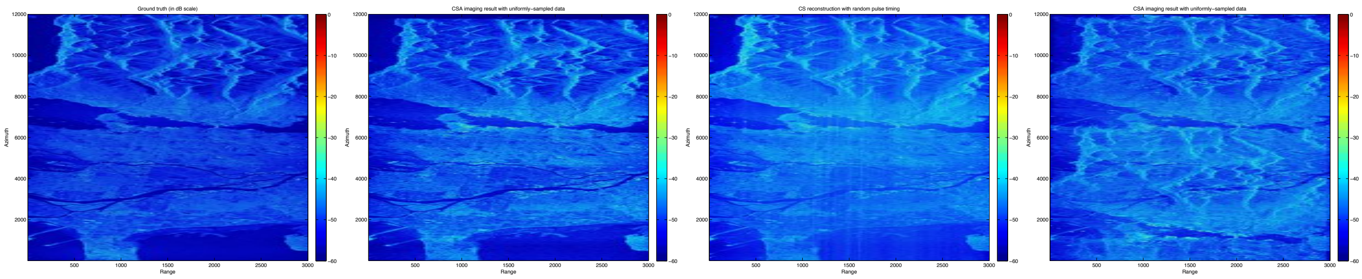
5. EXPERIMENTS

To test our approach, we conducted synthesized simulations on strip-map SAR, using parameters similar to the instrument on the RADARSAT-1 satellite [1]. We simulated the acquisition of a high-resolution scene, reconstructed from real SAR data, using classical SAR acquisition and the proposed randomized method. For the random pulsing, we use $\alpha = 6$ and $\beta = 0.5$, i.e., pulse timings are uniformly at random selected from 6 times the original PRF, with selection ratio of 50%, yielding an average PRF which is 3 times the original one.

To perform image formation, we use the algorithm described in Sec. 4 with K=100 iterations. We set $\epsilon_N(\mathbf{c})$ a small number such that for each iteration the top 70% largest wavelet coefficients are retained. For comparison, we show the results of a large area in Fig. 3, including (a) the true high-resolution ground reflectivity, (b) the reconstructed ground reflectivity using CSA and classical pulse timing with low PRF and low Doppler bandwidth to avoid azimuth aliasing, and (c) the reconstructed ground reflectivity using iterative reconstruction and randomized pulse timing, at average PRF $3\times$ the PRF possible with classical uniform pulse timing, and a corresponding increase in the Doppler bandwidth. For comparison, Fig. 3(d) demonstrates the significant azimuth aliasing introduced using classical pulse timing at low PRF but a high Doppler bandwidth, which makes the reconstructed image unusable. To demonstrate the improved resolution in Fig. 3(c) over (b), we zoom in the images and present some example regions in Fig. 4. We observe that the azimuth resolution is enhanced by the random chirp timing scheme and the enlarged Doppler bandwidth. Our results are consistent in a variety of experiments and experimental conditions.

6. DISCUSSION

In this paper we describe a randomized pulse timing scheme designed to overcome a fundamental trade-off between the azimuth resolution and the range coverage of classical SAR systems, which



(a) True ground reflectivity (b) Low PRF classical timing, small Doppler bandwidth (c) Randomized pulse timing, large Doppler bandwidth (d) Low PRF classical timing, large Doppler bandwidth

Fig. 3. Experimental results. Imaging a large area using classical vs. randomized pulse timing. Increasing the resolution by increasing the Doppler bandwidth in classical pulse timing results to aliasing due to the low PRF. Aliasing is eliminated with randomized pulse timing.

rely on uniform pulse timing. Our method, inspired by compressive sensing approaches, uses an iterative non-linear reconstruction algorithm for image formation. Experimental results on synthesized strip-map SAR data demonstrate that using our proposed method we can achieve high azimuth resolution without losing range coverage.

7. REFERENCES

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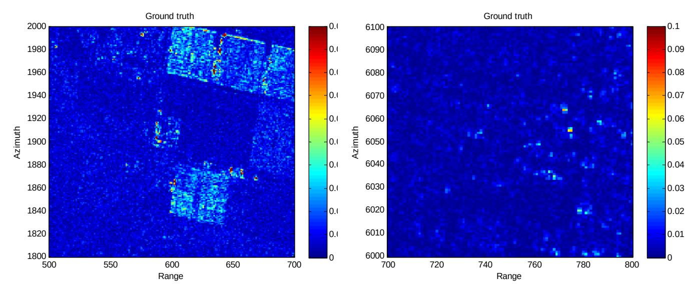
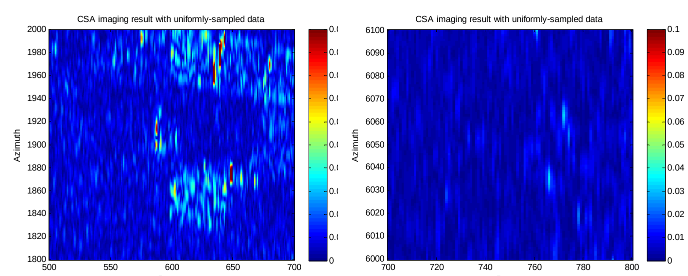
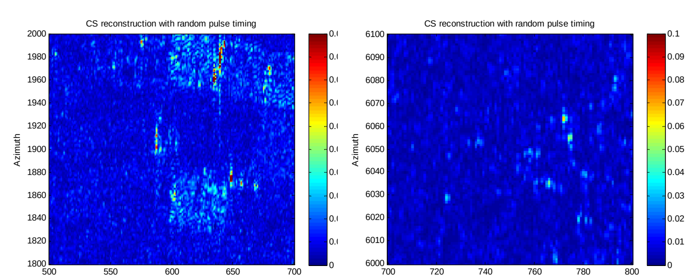


(a) True ground reflectivity



(b) Low PRF classical imaging, small Doppler bandwidth



(c) Randomized pulse timing, large Doppler bandwidth

Fig. 4. Experimental results of example regions. Randomized pulse timing produces significantly improved azimuth resolution.